A Parallel Mixed Integer Programming-Finite Element Method Technique for Global Design Optimization of Power Transformers

Eleftherios I. Amoiralis[1], Marina A. Tsili[2], Pavlos S. Georgilakis[1], Antonios G. Kladas[2], and Athanassios T. Souflaris[3]

[1]Department of Production Engineering and Management, Technical University of Crete, GR-73100 Chania, Greece
[2]Faculty of Electrical and Computer Engineering, National Technical University of Athens, GR-15780 Athens, Greece
[3]Schneider Electric AE, Elvim Plant, GR-32011 Inofyta, Greece

Transformer design optimization is determined by minimizing the transformer cost taking into consideration constraints imposed both by international specifications and customer needs. The main purpose of this work is the development and validation of an optimization technique based on a parallel mixed integer nonlinear programming methodology in conjunction with the finite element method, in order to reach a global optimum design of wound core power transformers. The proposed optimization methodology has been implemented into software able to provide a global feasible solution for every given set of initial values for the design variables, rendering it suitable for application in the industrial transformer design environment.

***Index Terms*—Design methodology, finite element methods, mixed integer programming, optimization methods, power transformers.**

I. Introduction

TRANSFORMER design optimization seeks a constrained minimum cost solution by optimally setting the transformer geometry parameters and the relevant electrical and magnetic quantities. The difficulty in achieving the optimum balance between the transformer cost and performance is becoming even more complicated nowadays, as the manufacturing materials (copper, aluminum, steel) are highly variable stock exchange commodities. Techniques that include mathematical models employing analytical formulas, based on design constants and approximations for the calculation of the transformer parameters are often the base of the design process adopted by transformer manufacturers [1]. However, the relevant technical literature comprises a variety of other approaches in order to cope with the complex problem of transformer design optimization, based on stochastic optimization methods such as genetic algorithms (GAs) that have been used for power transformer cost minimization [2], performance optimization of cast-resin distribution transformers with stack core technology [3] or toroidal core transformers [4]. The computational complexity of stochastic methods becomes quite considerable in case of the numerous iterations that may be required in order to achieve overall transformer optimization, therefore limiting the application of such methods in certain aspects of transformer design, as in [5] and [6], where artificial intelligence techniques are used for winding material selection and prediction of transformer losses and reactance, respectively, or [7], where particle swarm optimization is applied for the transformer thermal parameters estimation. Moreover, the optimality of the solution provided by GAs and other stochastic methods cannot be guaranteed [8] and multiple runs may result to different suboptimal solutions, with a significant difference between the worst and the best one. On the other hand, deterministic methods may provide more robust solutions to the transformer design optimization problem. In this context, the deterministic method of geometric programming has been proposed in [9] in order to deal with the design optimization problem of both low-frequency and high-frequency transformer. An important improvement in deterministic design optimization methods can be implemented by the incorporation of numerical methods, and various approaches have been developed [10], [11].

During the formulation of the transformer optimization problem, it is important to note that all the design variables can assume not only continuous values but also integer values (e.g., number of winding turns), thus, mixed integer programming (MIP) techniques [12] are very suitable for such problems. Moreover, there may be a number of suboptimal solutions to the problem, and the possibility of convergence to local optima has to be avoided. A method to locate the global optimum among the various local optima should therefore be adopted, with the least possible computational effort.

The present paper introduces the application of MIP to the transformer design optimization, developing a novel parallel implementation of MIP linked to finite element method (FEM), enhancing convergence to the global optimal solution. The novelty of the proposed method is based on the adoption of a 3-D cost-effective transformer FEM model that is directly linked to the proposed parallel MIP technique, ensuring its ability to reach a global optimum while maintaining low execution times.

The paper is organized as follows. Section II provides the mathematical background of the MIP method and the implementation details of the novel parallel MIP technique. Section III presents the transformer FEM model along with its importance for the convergence characteristics of the proposed methodology. Section IV presents the results from the application of the method to different actual transformer designs. These results prove the robustness and the efficiency of the proposed method in the solution of the transformer design optimization problem. Section V concludes the paper.

Digital Object Identifier 10.1109/TMAG.2007.915119

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

II. Proposed Methodology

A. Description of the Optimization Model

In order to find the global optimum of a constrained multi-variable function, MIP implements the Branch and Bound (BB) algorithm [13]. The standard form of a nonlinear objective function $\underline{f}(\underline{x})$ with n design variables x_j to be minimized by MIP is

$$\min \underline{f}(\underline{x}) = \min_{\underline{x}} \sum_{j=1}^{n} c_j f_j(\underline{x}) \tag{1}$$

subject to the following constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad i = 1, 2, \ldots, m \tag{2}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \tag{3}$$

$$x_j \in \mathcal{N} \text{ for all or some } j = 1, 2, \ldots, n \tag{4}$$

$$lb_j \leq x_j \leq ub_j, \qquad j = 1, 2, \ldots, n \tag{5}$$

where $\underline{f}$ is a $n \times 1$ matrix of objective functions f_j, $\underline{c}$ is a $n \times 1$ matrix of objective function coefficients c_j, $\underline{x}$ is a $n \times 1$ matrix of design variables x_j, $\underline{a}$ is a $m \times n$ matrix of constraints coefficients, $\underline{b}$ is a $m \times 1$ matrix of the upper values of the constraints, $\mathcal{N}$ refers to the set $\{0, 1, \ldots\}$, and $\underline{lb}$ and $\underline{ub}$ are $n \times 1$ matrices of lower and upper bounds on $\underline{x}$, respectively.

The BB method solves MIP by solving a sequence of nonlinear programming problems obtained by relaxing integrality conditions and including additional constraints. The number of additional constraints increases as the BB procedure progresses. These constraints separate the feasible region into complementary subregions. In particular, the BB method initially sets up lower and upper bounds for the optimal solution (objective function optimal value). The branching strategy iteratively decreases the upper bound and increases the lower bound. The difference between those bounds is a measure of the proximity of the current solution to the optimal solution if it exists. When minimizing, a lower bound for the optimal solution can be found by relaxing the integrality constraints of the original MIP and solving the resulting MIP. Analogously, the objective function value for any solution (satisfying integrality conditions) of the original MIP is the upper bound of the optimal solution.

This paper proposes a parallel MIP methodology in conjunction with FEM that operates as follows (Fig. 1): at the first stage, the upper and lower bounds of the design vector are selected in accordance with the transformer rated power, defining the interval of the design variables. Afterward, a set of subintervals for the design variables is generated (randomly by software implementation of the proposed methodology or manually by the user of that software), and distributed into q parallel implementations of MIP. The initial values of each MIP derive from the mean value of the selected bounds, while the transformer parameters are calculated by an algebraic design model, based on analytical equations. In doing so, local optima are avoided. The FEM-based cycle of iterations uses the parallel MIP solution as initial vector and converges to global optimum. In this procedure, the solution does not depend on the initial values of the design vector, and there is no need for fine-tuning the convergence parameters of the optimization algorithm. The parallel implementation of the MIP cycles reduces the respective computational time by a factor at least equal to $1/q$. Moreover, it enhances the overall process by feeding the last cycle a tentative solution near the optimal one, thus reducing the number of the more time consuming FEM-based iterations. Furthermore, other difficulties in the statement of the problem and its adaptation to each considered transformer, which may exist during the application of stochastic optimization methods, are bypassed.

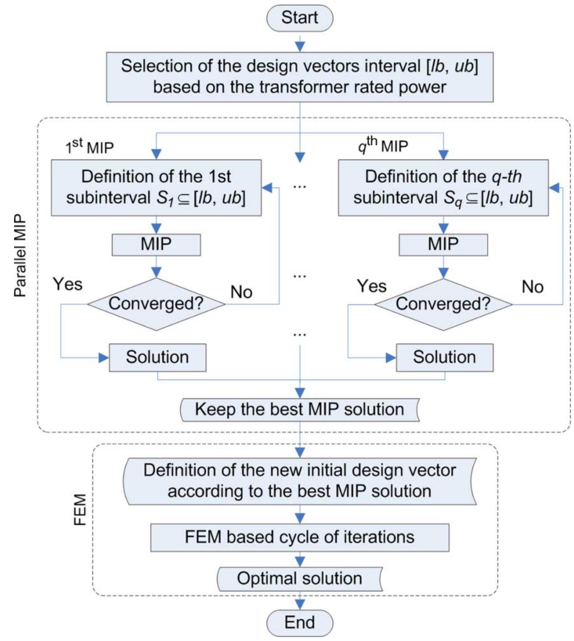

Fig. 1. Flowchart of the proposed technique.

B. Mathematical Formulation of the Transformer Design Optimization Problem

The goal of the proposed parallel mixed integer optimization process is to find a set of integer variables linked to a set of continuous variables that minimize the objective function (active part cost) and meet the restrictions imposed on the transformer design problem. Under the previous definitions, a mixed integer nonlinear problem for optimizing the transformer design is based on the minimization of the cost of the transformer active part

$$\min \underline{f}(\underline{x}) = \min_{\underline{x}} \sum_{j=1}^{4} c_j f_j(\underline{x}) \tag{6}$$

where c_1 is the primary winding unit cost (Euros per kilogram), f_1 is the primary winding weight (kilograms), c_2 is the secondary winding unit cost (Euros per kilogram), f_2 is the secondary winding weight (kilograms), c_3 is the magnetic material unit cost (Euros per kilogram), f_3 is the magnetic material weight (kilogram), c_4 is the insulating paper unit cost (Euros per kilogram), f_4 is the insulating paper weight (kilogram), and $\underline{x}$ is the vector of the five design variables, namely the number of low-voltage turns, the width of core leg (D), the core window height (G), the magnetic induction magnitude (B) and the core material type.

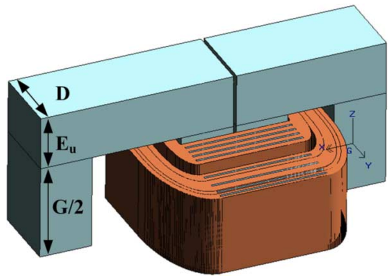

Fig. 2. FEM model and core constructional parameters (G: height of the core window, D: width of the core leg, Eu: thickness of the core leg).

The minimization of the cost of the transformer active part is subject to the following constraints that are based on the tolerances specified by IEC 60076-1 [14].

$$DNLL + DLL - 1.10 \cdot (GNLL + GLL) < 0 \tag{7}$$
$$DNLL - 1.15 \cdot GNLL < 0 \tag{8}$$
$$DLL - 1.15 \cdot GLL < 0 \tag{9}$$
$$0.90 \cdot GU < DU < 1.10 \cdot GU \tag{10}$$

where

$DNLL$ designed no-load losses (W)
DLL designed load losses (W)
DU designed short-circuit impedance (%).
$GNLL$ guaranteed no-load losses (W)
GLL guaranteed load losses (W)
GU guaranteed short-circuit impedance (percent).

III. Integration of FEM to the Optimization Model

A. Transformer FEM Model

Fig. 2 illustrates the FEM model adopted in the development of the optimization technique [15]. It comprises the low- and high-voltage windings of one phase, as well as the small and large iron core that surrounds them. Due to the symmetries of the problem, the model is reduced to one fourth of the device. The integration of the FEM model into the optimization algorithm is realized as follows (Fig. 1): the optimal solution provided by the parallel MIP method is used as the initial vector for the design variables and a new cycle of iterations is performed, reducing the design variables only to the continuous ones (the remaining integer variables are considered equal to the optimal value provided by the previous iterations). During this cycle, FEM is used for the calculation of the transformer characteristics.

B. FEM Contribution to Global Optimum Convergence

Although the proposed parallel MIP methodology is particularly suitable for the transformer design optimization, difficulties in its convergence may be experienced, a problem that is well known in deterministic algorithms during the solution of nonlinear optimization problems with multiple local optima.

TABLE I
Technical Characteristics of the Considered Transformers

Rated power (kVA)	Primary / secondary voltages (kV)	Guaranteed load loss (W)	Guaranteed no-load loss (W)	Guaranteed short-circuit impedance (%)
160	20 / 0.4	2350	300	4
250	11 / 0.4	3250	530	4
400	20 / 0.4	4600	610	4
630	21 / 0.42	8700	1200	6
1000	21 / 0.42	13000	1700	6
1600	20 / 0.4	20000	2600	6

TABLE II
Cost Comparison of Optimum Designs Provided by the Proposed and the Current Transformer Design Methodology

Rated power (kVA)	Cost of the proposed method (€)	Cost of the current method (€)	Cost reduction achieved by the proposed method (%)
160	1865	1894	1.52
250	2268	2369	4.27
400	3588	3810	5.83
630	3692	3759	1.80
1000	4948	5185	4.57
1600	7682	8140	5.62
		Average	**3.94**

These difficulties are likely to arise in cases of designs with special requirements, where compatibility to the constraints may direct the method to seek for the optimum in subintervals where there is no feasible solution, eliminating the possibility of convergence. However, the integration of FEM to the optimization overcomes this possibility, ensuring convergence to the global optimum in any case. This ability renders FEM an essential part of the proposed method, since the lack of the FEM-based cycle of iterations may result to inability to find a feasible solution or local optimal traps.

IV. Results and Discussion

The robustness of the proposed methodology is presented in comparison with that of current methodology [1] that is already applied in a transformer manufacturing industry. The proposed method minimizes the cost of the transformer active part, (6), subject to the constraints (7)–(10) by seeking the optimum settings of the five design variables, namely, the core constructional parameters D and G shown in Fig. 2 (continuous variables), the magnetic induction (continuous variable), the type of the core magnetic steel, namely M4 0.27, MOH 0.27, MOH 0.23, ZDMH90 0.23 (each of the four types are represented by an integer identification number), and the number of turns (integer variable). Six actual transformer designs are considered, using different loss categories, according to CENELEC (Harmonization document, HD 428.1 S1:1992). The common technical characteristics of these six designs are: the material of the external and internal coil is copper, the vector group is Dyn11 apart from 1600 kVA that is Dyn5, and the operating frequency is 50 Hz. Table I shows the technical characteristics of each transformer. Table II compares the cost of each optimal transformer design obtained a) by the proposed method, and b) by the current method [1] that is based on a heuristic optimization technique. Table II clearly illustrates that the proposed method converges to an optimum solution that has on average a 3.94% lower cost than the current method used by the manufacturer.

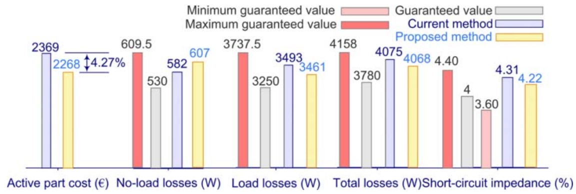

Fig. 3. Comparison of characteristics of the optimum designs provided by the proposed and the current design method, for the 250-kVA transformer.

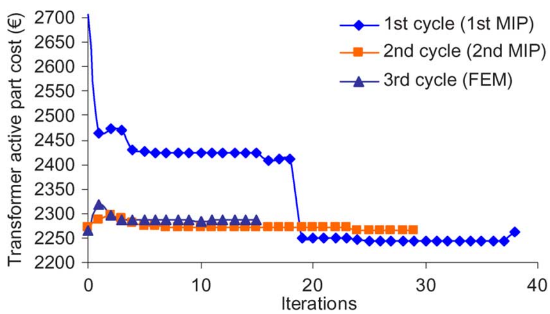

Fig. 4. Convergence history of the proposed technique in the case of the 250-kVA transformer design optimization.

The mean time needed for the convergence of the proposed algorithm to the solutions of Table II is 15 min in an average performance computer (80% of this time is consumed on the FEM-based cycle of iterations). Apart from the better convergence characteristics of the proposed algorithm, the difference between the optimum values is due to the difference in the permissible range of the design variables used in the heuristic algorithm, which is confined to discrete steps of the variables instead of the complete interval $[\underline{lb}\ \underline{ub}]$.

Fig. 3 presents the comparison of technical characteristics of the optimum designs provided by the proposed and the current method, for the 250-kVA transformer. The optimal design vector of the proposed method, for this transformer is: number of low-voltage turns $= 18$, $D = 250$ mm, $G = 250$ mm, $B = 16620$ Gauss and the type of core material is MOH 0.27.

Fig. 4 shows the convergence history of the proposed method for the 250-kVA transformer, concerning the active part cost, resulting in an optimum (minimum) cost of 2268 Euros provided by the FEM cycle (third cycle), after $q = 2$ parallel MIP cycles, where the first and the second cycle correspond to the two parallel MIP cycles. The optimum value provided by the FEM cycle appears to be slightly worse than the one provided by the two parallel MIP cycles, due to the errors of the simplified analytical formulas of the design model employed in MIP cycle. These errors are overcome by the enhanced accuracy provided by FEM, resulting to the convergence to the global optimum.

V. Conclusion

In this paper, a parallel MIP technique using the BB algorithm and FEM method is proposed for the solution of the power transformer design optimization problem. The proposed method is very effective because of its robustness, its high execution speed and its ability to effectively search the large solution space. The difficulty of convergence to a global optimum often encountered by deterministic methods during the solution of nonlinear problems is overcome by the proposed finite element incorporation to the parallel MIP model. Moreover, the global optimum obtained by the proposed method is not satisfactorily approached by continuous variable optimization techniques. The validity of the proposed method is clearly illustrated by its application to a wide spectrum of actual transformers, of different voltage ratings and loss categories, resulting to optimum designs with an average cost saving of 3.94% in comparison with the existing method used by a transformer manufacturer. The application of the proposed parallel MIP-FEM method can be generalized to the optimum design of other electric machines.

Acknowledgment

This work was supported in part by the 03ED045 Research Project that is cofinanced by EU-European Social Fund (75%) and the Greek Ministry of Development-GSRT (25%).

References

[1] P. S. Georgilakis, M. A. Tsili, and A. T. Souflaris, *J. Mater. Proc. Tech.*, vol. 181, no. 1–3, pp. 260–266, 2007.

[2] L. Hui, H. Li, H. Bei, and Y. Shunchang, in *Proc. 5th Int. Conf. Electr. Machines Syst.*, 2001, vol. 1, pp. 242–245.

[3] S. Elia, G. Fabbri, E. Nistico, and E. Santini, in *SPEEDAM 2006*, pp. 1473–1477.

[4] N. Tutkun and A. Moses, *J. Magn. Magn. Mater.*, vol. 277, no. 1–2, pp. 216–220, 2004.

[5] E. I. Amoiralis, P. S. Georgilakis, T. D. Kefalas, M. A. Tsili, and A. G. Kladas, *IEEE Trans. Magn.*, vol. 43, no. 4, pp. 1633–1636, 2007.

[6] L. H. Geromel and C. R. Souza, in *IEEE Can. Conf. Electrical and Computer Engineering*, 2002, vol. 1, pp. 285–290.

[7] W. H. Tang, S. He, E. Prempain, Q. Wu, and J. Fitch, in *Proc. IEEE Congr. Evolutionary Computation*, Sep. 2005, vol. 3, pp. 2745–2751.

[8] S. A. Kazarlis, A. G. Bakirtzis, and V. Petridis, *IEEE Trans. Power Syst.*, vol. 11, no. 1, pp. 83–92, Feb. 1996.

[9] R. Jabr, *IEEE Trans. Magn.*, vol. 41, no. 11, pp. 4261–4269, 2005.

[10] T. H. Pham, S. J. Salon, and S. R. H. Hoole, *IEEE Trans. Magn.*, vol. 32, no. 5, pp. 4287–4289, 1996.

[11] O. A. Mohammed and W. K. Jones, *IEEE Trans. Magn.*, vol. 26, no. 2, pp. 666–669, 1990.

[12] L. A. Wolsey, *Integer Programming*. Hoboken, NJ: Wiley, 1998.

[13] E. Castillo, A. J. Conejo, P. Pedregal, R. Garcia, and N. Alguacil, *Building and Solving Mathematical Programming Models in Engineering and Science*. Hoboken, NJ: Wiley, 2002.

[14] IEC 60076-1, Power Transformers—Part 1: General 2000.

[15] M. A. Tsili, A. G. Kladas, P. S. Georgilakis, A. T. Souflaris, and D. G. Paparigas, *Elec. Power Syst. Res.*, vol. 76, pp. 729–741, 2006.

Manuscript received June 22, 2007. Corresponding author: A. G. Kladas (e-mail: kladasel@central.ntua.gr).